Thévenin's Theorem

Thévenin's theorem is extremely useful in the analysis of complex electrical circuits. It states that any network that has two accessible terminals, A and B, can be replaced, as far as its external behavior is concerned, by a single emf acting in series with a single resistance between A and B. The single equivalent emf is that emf measured across A and B when the circuit external to the network is disconnected. The single equivalent resistance is the resistance of the network when all current and voltage sources within it are reduced to zero. To calculate this internal resistance of the network, all current sources within it are treated as open circuits and all voltage sources as short circuits. The proof of Thévenin's theorem can be found in Skilling (1967).

Figure A2.1 shows part of a network consisting of a voltage source and four resistances. As far as its behavior external to terminals A and B is concerned, this can be regarded as a single voltage source, V_t, and a single resistance, R_t. Applying Thévenin's theorem, R_t is found first of all by treating V_1 as a short circuit, as shown in Figure A2.2. These are simply two resistances, R_1 and $(R_2 + R_4 + R_5)$, in parallel. The equivalent resistance, R_t, is thus given by

$$R_t = \frac{R_1(R_2 + R_4 + R_5)}{R_1 + R_2 + R_4 + R_5},$$

where V_t is the voltage drop across AB. To calculate this, it is necessary to carry out an intermediate step of working out the current flowing, I. Referring to Figure A2.1, this is given by

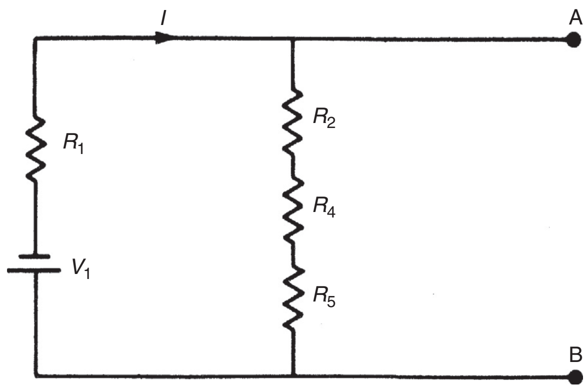

Figure A2.1

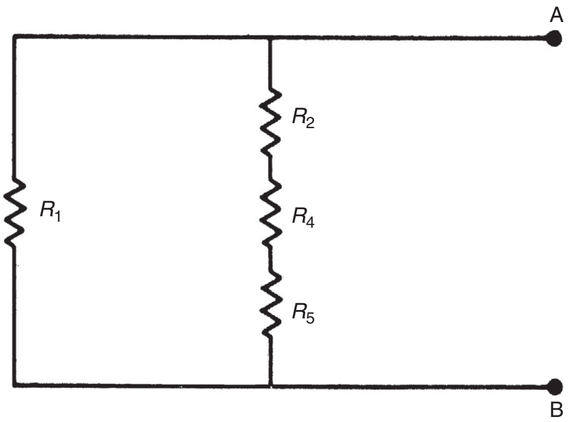

Figure A2.2

$$I = \frac{V_1}{R_1 + R_2 + R_4 + R_5}.$$

Now, V_t can be calculated from

$$\begin{aligned} V_t &= I(R_2 + R_4 + R_5) \\ &= \frac{V_1(R_2 + R_4 + R_5)}{R_1 + R_2 + R_4 + R_5}. \end{aligned}$$

The network of Figure A2.1 has thus been reduced to the simpler network shown in Figure A2.3.

Let us now proceed to the typical network problem of calculating the current flowing in resistor R_3 of Figure A2.4. R_3 can be regarded as an external circuit or load on the rest of the network consisting of V_1, R_1, R_2, R_4, and R_5, as shown in Figure A2.5. This network of V_1, R_1, R_2, R_4, and R_5 is that shown in Figure A2.6. This can be rearranged to the network shown in Figure A2.1, which is equivalent to the single voltage source and resistance,

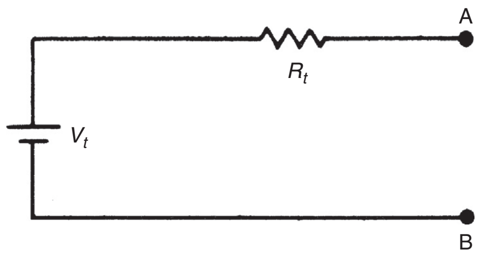

Figure A2.3

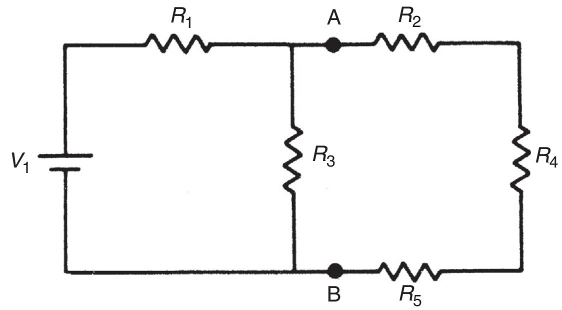

Figure A2.4

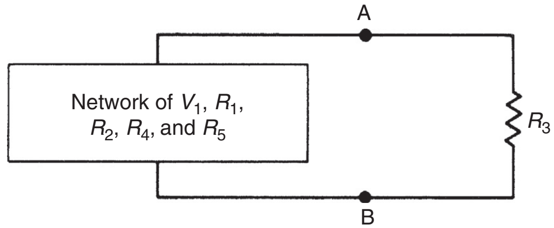

Figure A2.5

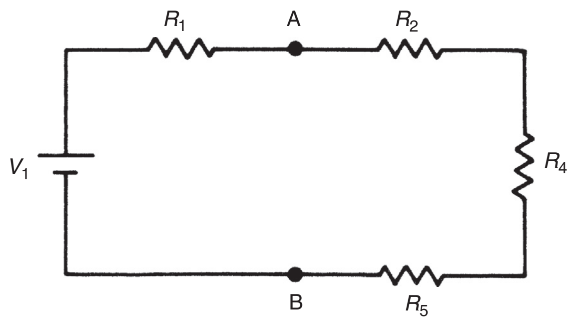

Figure A2.6

V_t and R_t, calculated earlier. The whole circuit is then equivalent to that shown in Figure A2.7, and the current flowing through R_3 can be written as

$$I_{AB} = \frac{V_t}{R_t + R_3}.$$

Thévenin's theorem can be applied successively to solve ladder networks of the form shown in Figure A2.8. Suppose, in this network, that it is required to calculate the current flowing in branch XY.

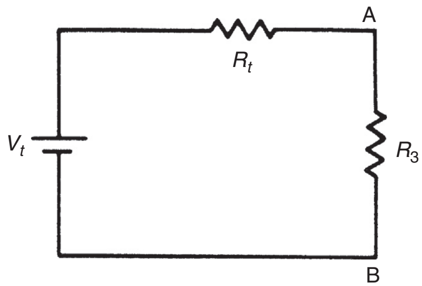

Figure A2.7

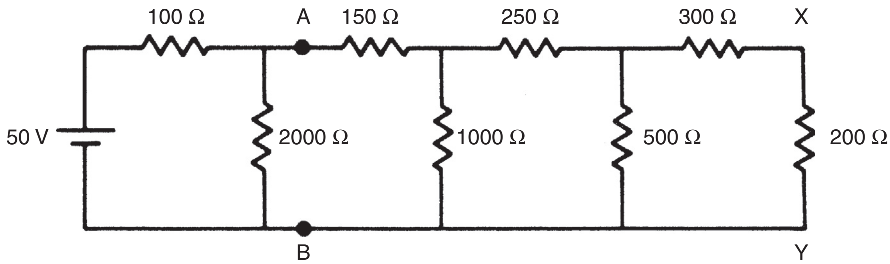

Figure A2.8

The first step is to imagine two terminals, A and B, in the circuit and regard the network to the right of AB as a load on the circuit to the left of AB. The circuit to the left of AB can be reduced to a single equivalent voltage source, E_{AB}, and resistance, R_{AB}, by Thévenin's theorem. If the 50-V source is replaced by its zero internal resistance (i.e., by a short circuit), then R_{AB} is given by

$$\frac{1}{R_{AB}} = \frac{1}{100} + \frac{1}{2000} = \frac{2000 + 100}{200,000}.$$

Hence,

$$R_{AB} = 95.24\ \Omega.$$

When AB is an open circuit, the current flowing round the loop to the left of AB is given by

$$I = \frac{50}{100 + 2000}.$$

Hence, E_{AB}, the open circuit voltage across AB, is given by

$$E_{AB} = I \times 2000 = 47.62 \text{ volts}.$$

We can now replace the circuit shown in Figure A2.8 by the simpler equivalent circuit shown in Figure A2.9.

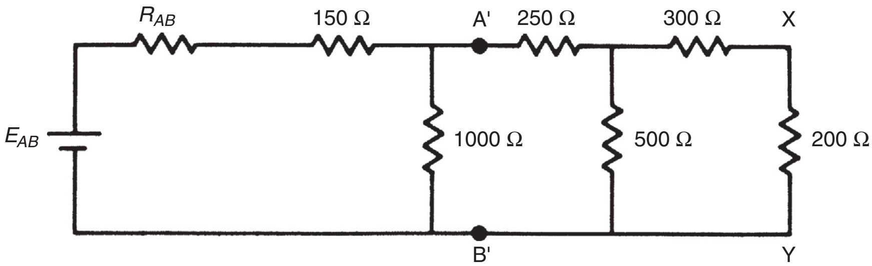

Figure A2.9

The next stage is to apply an identical procedure to find an equivalent circuit consisting of voltage source $E_{A'B'}$ and resistance $R_{A'B'}$ for the network to the left of points A′ and B′ in Figure A2.9:

$$\frac{1}{R_{A'B'}} = \frac{1}{R_{AB} + 150} + \frac{1}{1000} = \frac{1}{245.24} + \frac{1}{1000} = \frac{1245.24}{245,240}.$$

Hence,

$$R_{A'B'} = 196.94\ \Omega$$

$$E_{A'B'} = \frac{1000}{R_{AB} + 150 + 1000} \times E_{AB} = 38.24 \text{ volts.}$$

The circuit can now be represented in the yet simpler form shown in Figure A2.10. Proceeding as before to find an equivalent voltage source and resistance, $E_{A''B''}$ and $R_{A''B''}$, for the circuit to the left of A'' and B'' in Figure A2.10:

$$\frac{1}{R_{A''B''}} = \frac{1}{R_{A'B'} + 250} + \frac{1}{500} = \frac{500 + 446.94}{223,470}.$$

Hence,

$$R_{A''B''} = 235.99\ \Omega$$

$$E_{A''B''} = \frac{500}{R_{A'B'} + 250 + 500} E_{A'B'} = 20.19 \text{ volts.}$$

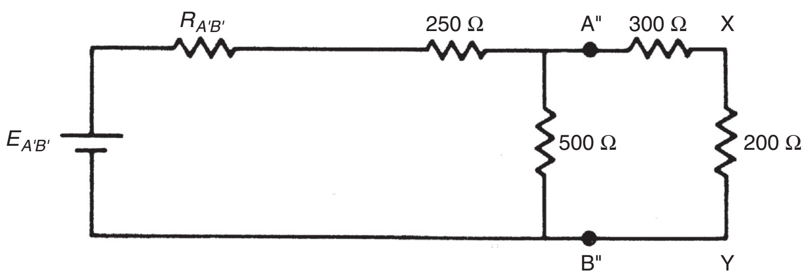

Figure A2.10

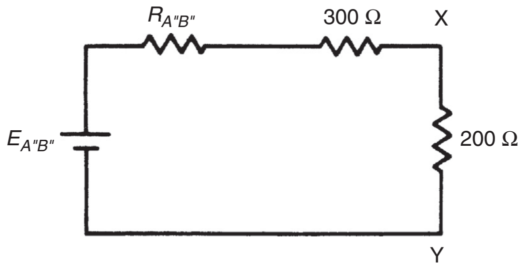

Figure A2.11

The circuit has now been reduced to the form shown in Figure A2.11, where the current through branch XY can be calculated simply as

$$I_{XY} = \frac{E_{A''B''}}{R_{A''B''} + 300 + 200} = \frac{20.19}{735.99} = 27.43 \text{ mA}.$$

Reference

Skilling, H.H., 1967. Electrical Engineering Circuits. Wiley, New York.